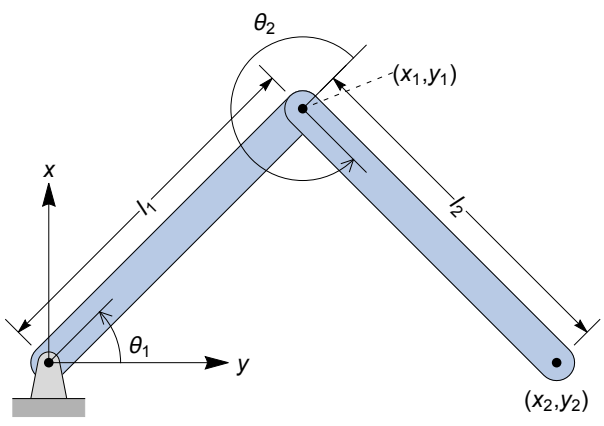
Two-Link Planar Robotic Arm

The two-link planar robotic arm is a frictionless, constrained mechanical system. The system consists of two links attached to each other through a joint, such that one end of the first link is held fixed, while the trajectory at the end of the second link is specified. The following figure shows a schematic of the system.



Equations of Motion

Equations of Motion describe the motions of parts of a system in terms of time, or as a function of time.

$$x_1(t) = l_1 \cos(\theta_1(t))$$
$$y_1(t) = l_1 \sin(\theta_1(t))$$
$$x_2(t) = l_1 \cos(\theta_1(t)) + l_2 \cos(\theta_1(t) + \theta_2(t))$$
$$y_2(t) = l_1 \sin(\theta_1(t)) + l_2 \sin(\theta_1(t) + \theta_2(t))$$

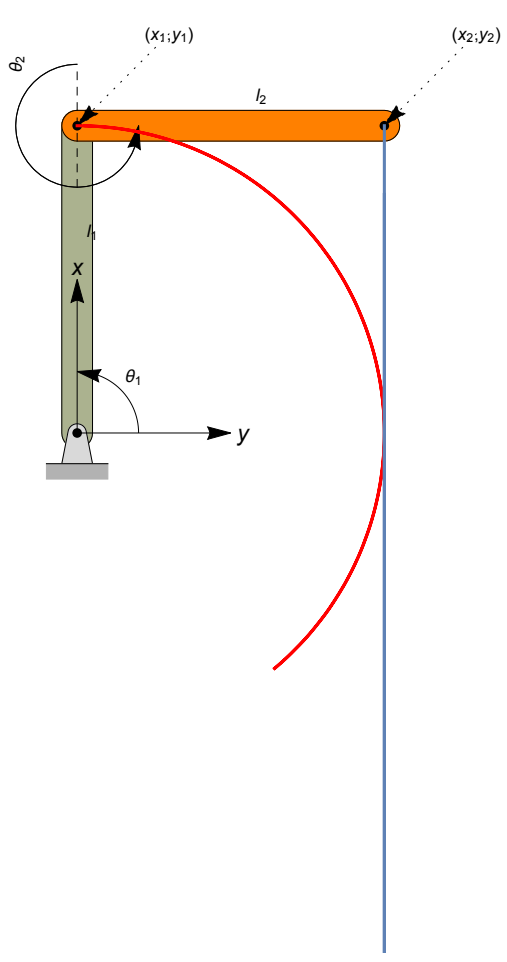
$x_1(t)$ stands for horizontal coordinate x_1 as function of time i.e. given a value time t, $x_1(t)$ is the value x_1 at time t.
$\theta_1(t)$ stands for angle θ_1 as function of time i.e. given a value time t, $\theta_1(t)$ is the value for angle θ_1 at time t.

Example

$l_1 = 1$
$l_2 = 1$
$x_1(t) = \cos(\theta_1(t))$
$y_1(t) = \cos(\theta_1(t) + \theta_2(t)) + \cos(\theta_1(t))$
$x_2(t) = \sin(\theta_1(t))$
$y_2(t) = \sin(\theta_1(t) + \theta_2(t)) + \sin(\theta_1(t))$

Initial configuration:

$t = 0$
$\theta_1(0) = 90.,\ \theta_2(0) = 270.$
$x_1(0) = 0,\ y_1(0) = 1.$
$x_2(0) = 1.,\ y_2(0) = 1.$



Blue curve is the constraint imposed on second arm's path.
Red curve is **always** an arc of radius $l_1 = 1$ for first arm's path.

At later time:

$t = 0.6$
$\theta_1(0.6) = 67.2955,\ \theta_2(0.6) = 240.585$
$x_1(0.6) = 0.385978,\ y_1(0.6) = 0.922508$
$x_2(0.6) = 1.,\ y_2(0.6) = 0.133219$