Solve $\mathrm{Cos}(\theta)=a$ for θ

Solve trigonometric equations without using any identities.

Find all θ satisfying the equation $\cos(\theta)=-\frac{1}{\sqrt{2}}$.

Step 1.

Since cosine value is negative, find a known first quadrant angle with cosine equal to the absolute value of the given cosine or $\frac{1}{\sqrt{2}}$

Step 1.a

$\theta=\frac{\pi}{4}$ where $\cos\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

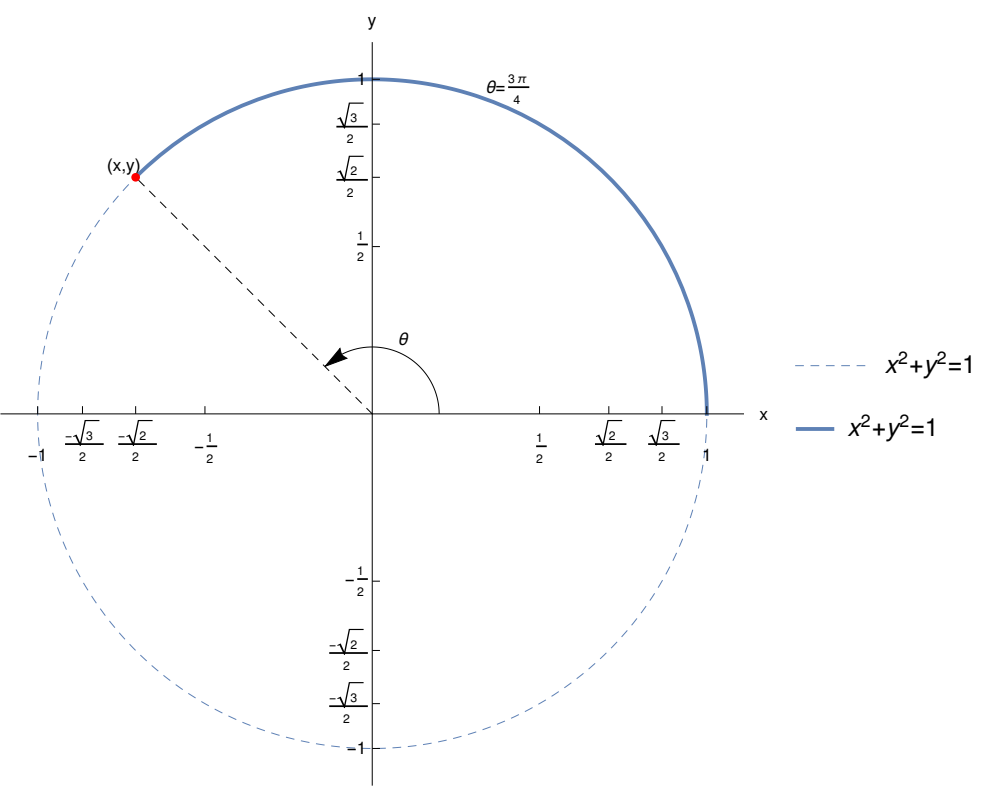
Step 1.b

Subtract this angle from π which causes the cosine to become negative

$\cos\left(\pi-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

Switch to $\pi-\frac{\pi}{4}=\frac{3\pi}{4}$

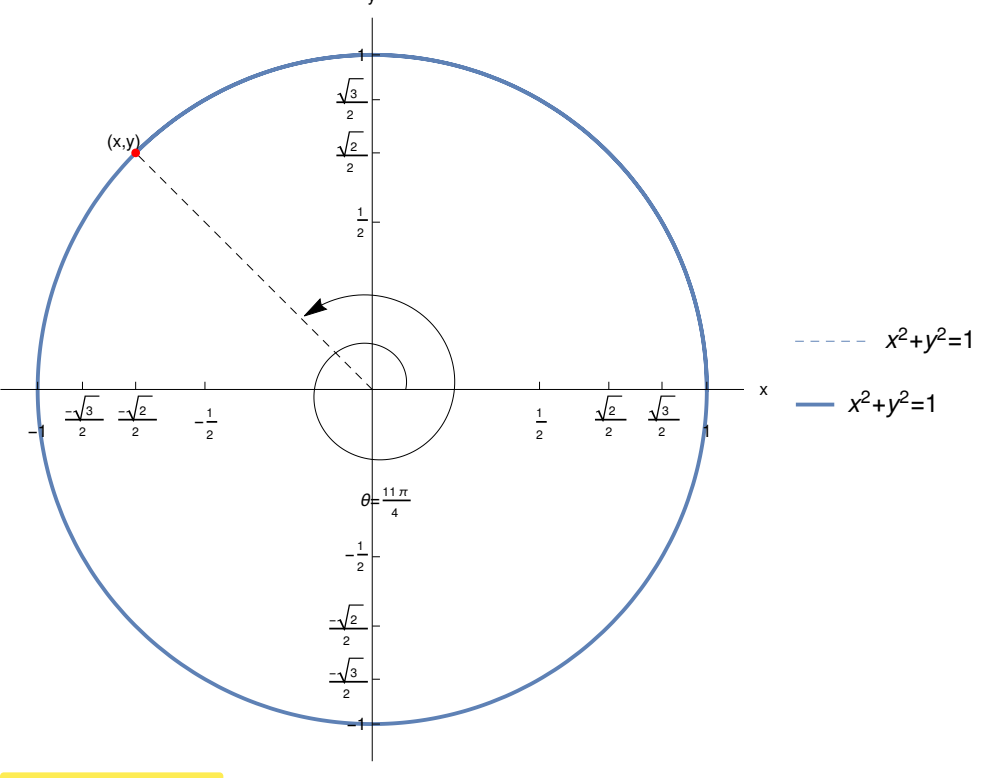
$\theta=\frac{3\pi}{4}$



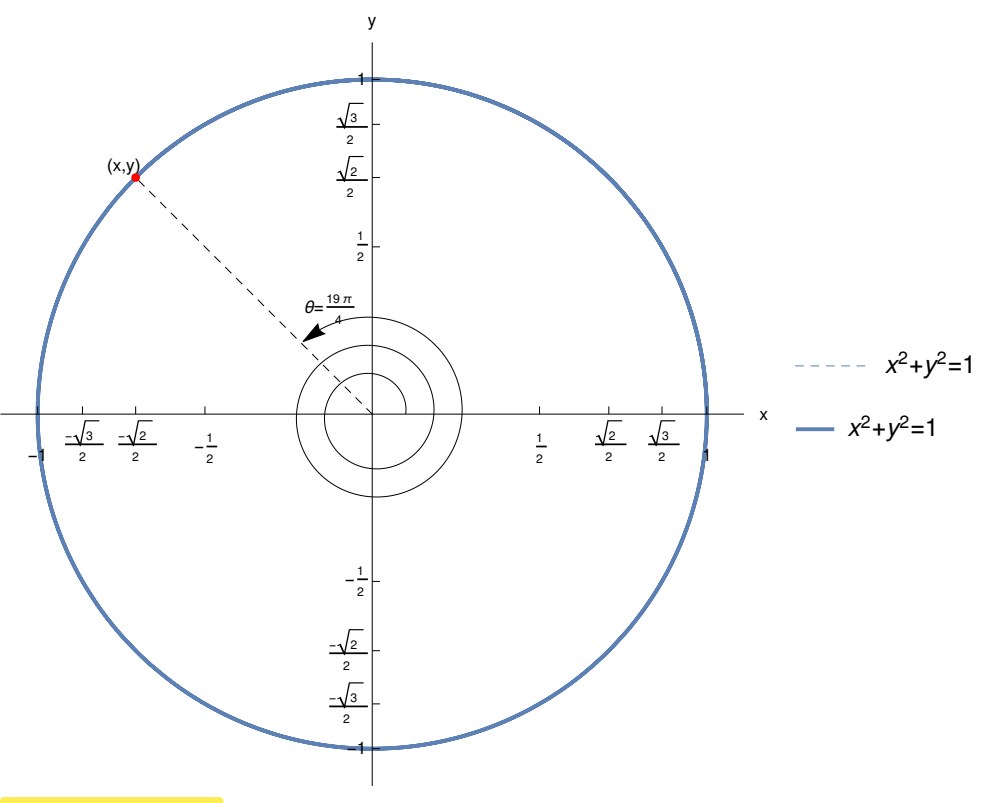
Step 2.

Add positive integer multiples of 2π

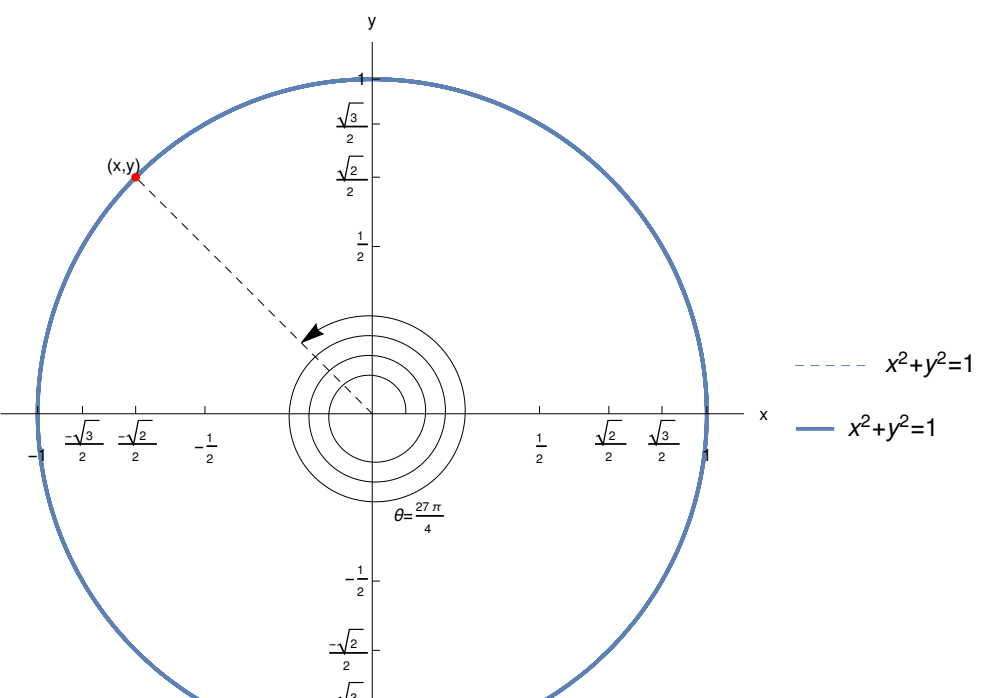
$\theta=2\pi\times 1+\frac{3\pi}{4}=\frac{11\pi}{4}$



$\theta=2\pi\times 2+\frac{3\pi}{4}=\frac{19\pi}{4}$

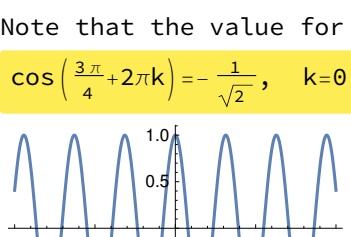


$\theta=2\pi\times 3+\frac{3\pi}{4}=\frac{27\pi}{4}$



Note that the value for cosine does not change.

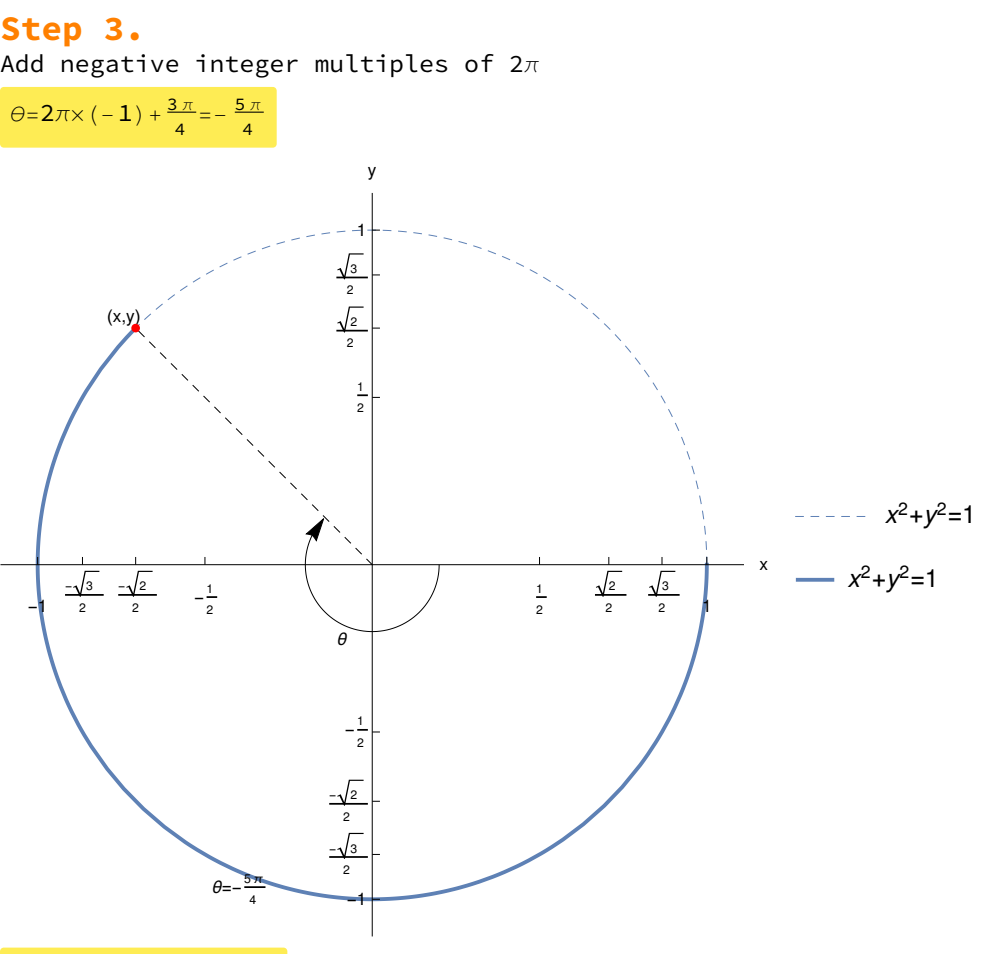
$\cos\left(\frac{3\pi}{4}+2\pi k\right)=-\frac{1}{\sqrt{2}},\quad k=0,\ 1,\ 2,\ \cdots\quad k\in\mathbb{Z}^{+}$



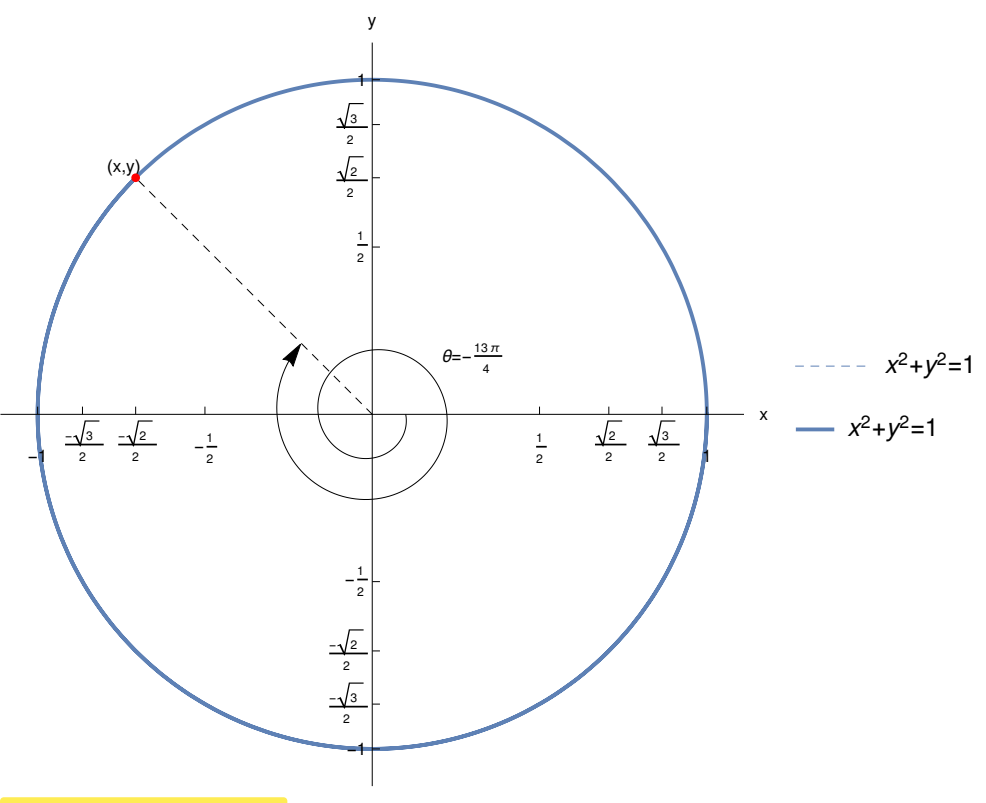
Step 3.

Add negative integer multiples of 2π

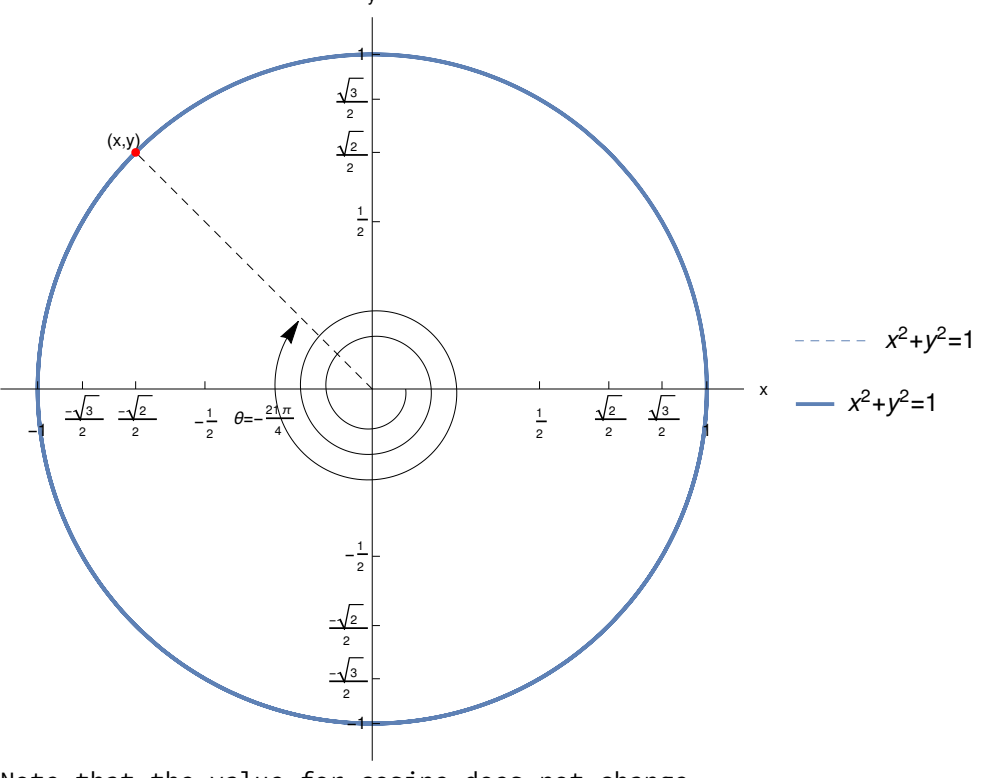
$\theta=2\pi\times(-1)+\frac{3\pi}{4}=-\frac{5\pi}{4}$



$\theta=2\pi\times(-2)+\frac{3\pi}{4}=-\frac{13\pi}{4}$



$\theta=2\pi\times(-3)+\frac{3\pi}{4}=-\frac{21\pi}{4}$

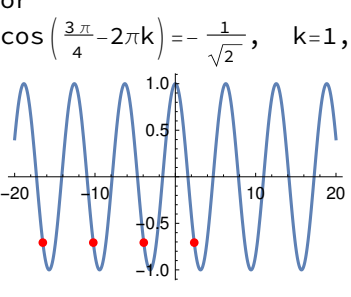


Note that the value for cosine does not change.

$\cos\left(\frac{3\pi}{4}+2\pi k\right)=-\frac{1}{\sqrt{2}},\quad k=-1,\ -2,\ \cdots\quad k\in\mathbb{Z}^{-}$

or

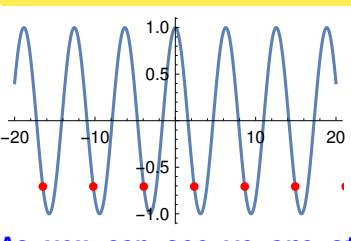
$\cos\left(\frac{3\pi}{4}-2\pi k\right)=-\frac{1}{\sqrt{2}},\quad k=1,\ 2,\ \cdots$



Step 4.

Combine the two solutions

$\cos\left(\frac{3\pi}{4}+2\pi k\right)=-\frac{1}{\sqrt{2}},\quad k=\cdots,\ -1,\ -2,\ 0,\ 1,\ 2\ \cdots\quad k\in\mathbb{Z}$



As you can see we are still missing half the solutions!

Step 5. $\mathrm{Cos}(-\theta)=a$

Repeat the above steps but multiply the starting angle by -1:

$\theta=-\left(\frac{3\pi}{4}\right)=-\frac{3\pi}{4}$

$\cos\left(-\frac{3\pi}{4}+2\pi k\right)=-\frac{1}{\sqrt{2}},\quad k=\cdots,\ -1,\ -2,\ 0,\ 1,\ 2\ \cdots\quad k\in\mathbb{Z}$