Solve Cos(θ)=a for θ

Solve trigonometric equations without using any identities.

Find all θ satisfying the equation $\cos(\theta)=-\frac{\sqrt{3}}{2}$.

Step 1.

Since cosine value is negative, find a known first quadrant angle with cosine equal to the absolute value of the given cosine or $\frac{\sqrt{3}}{2}$

Step 1.a

$\theta=\frac{\pi}{6}$ where $\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

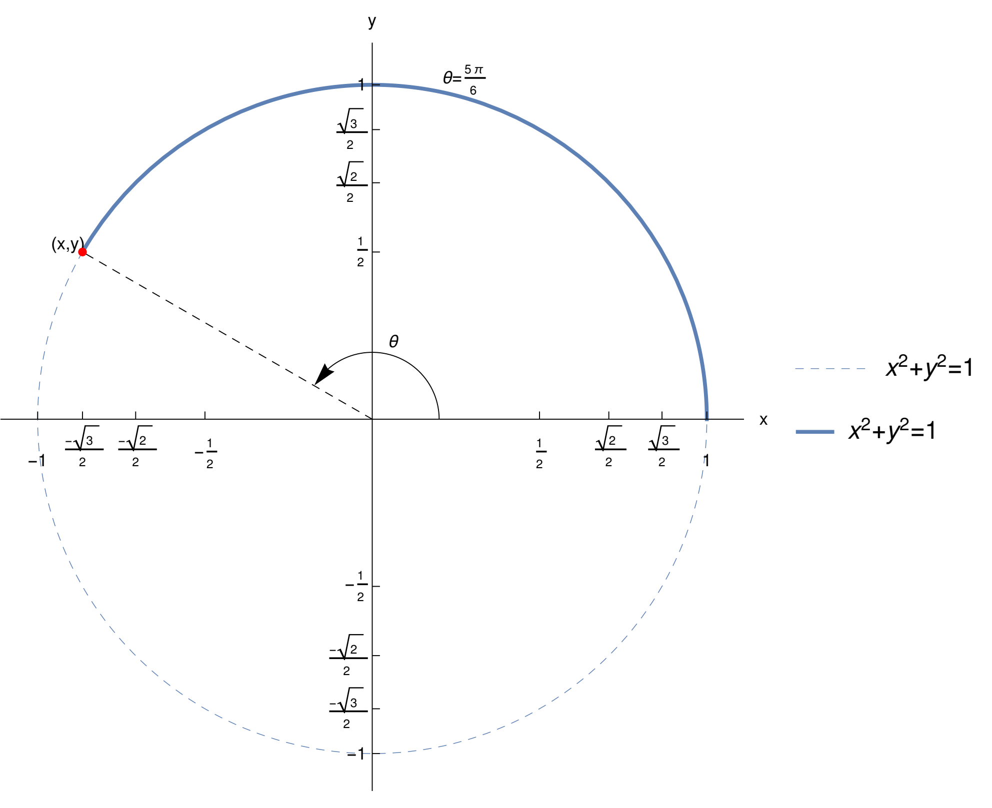
Step 1.b

Subtract this angle from π which causes the cosine to become negative

$\cos\left(\pi-\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{2}$

Switch to $\pi-\frac{\pi}{6} = \frac{5\pi}{6}$

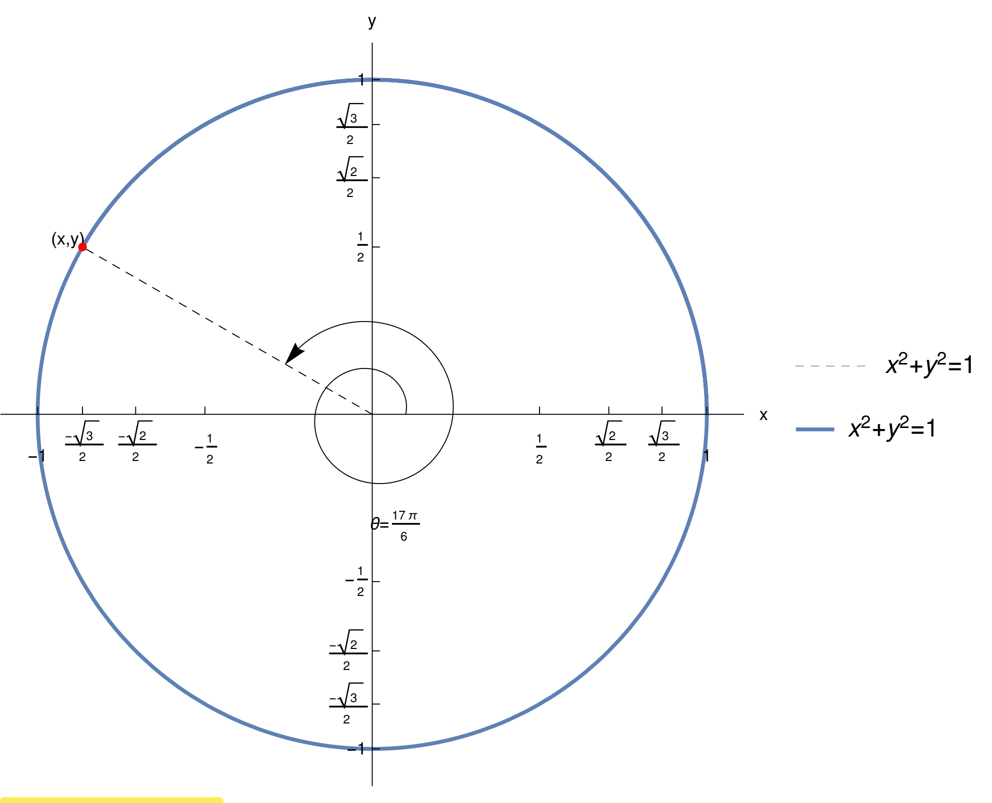
$\theta=\frac{5\pi}{6}$



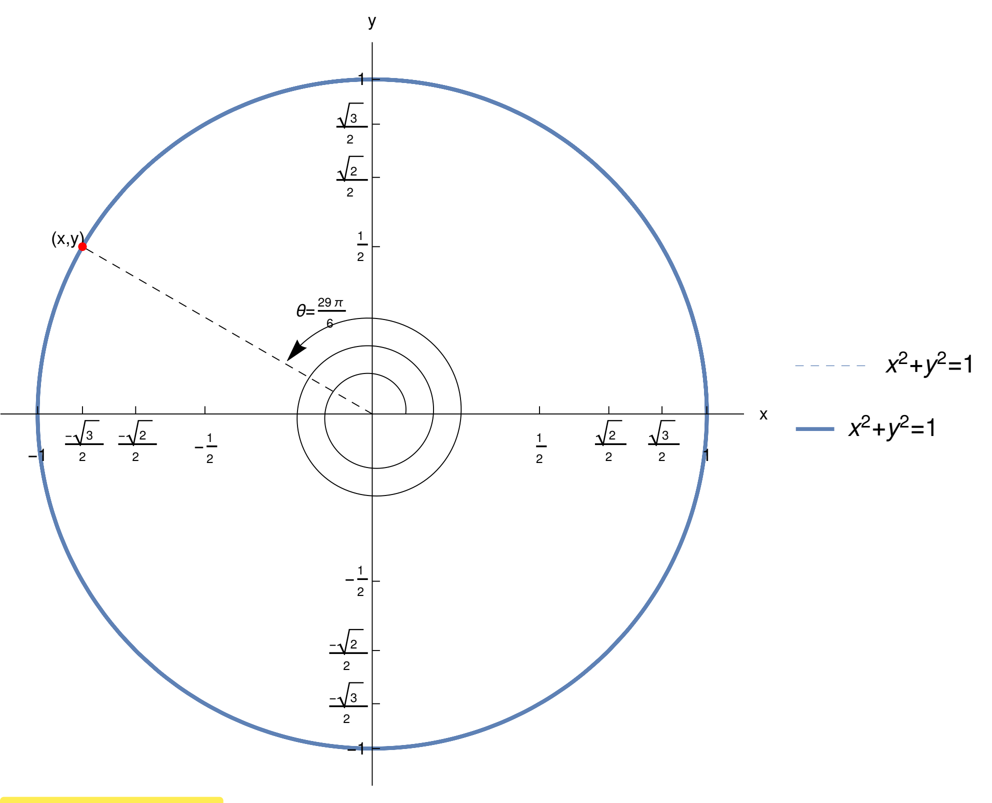
Step 2.

Add positive integer multiples of 2π

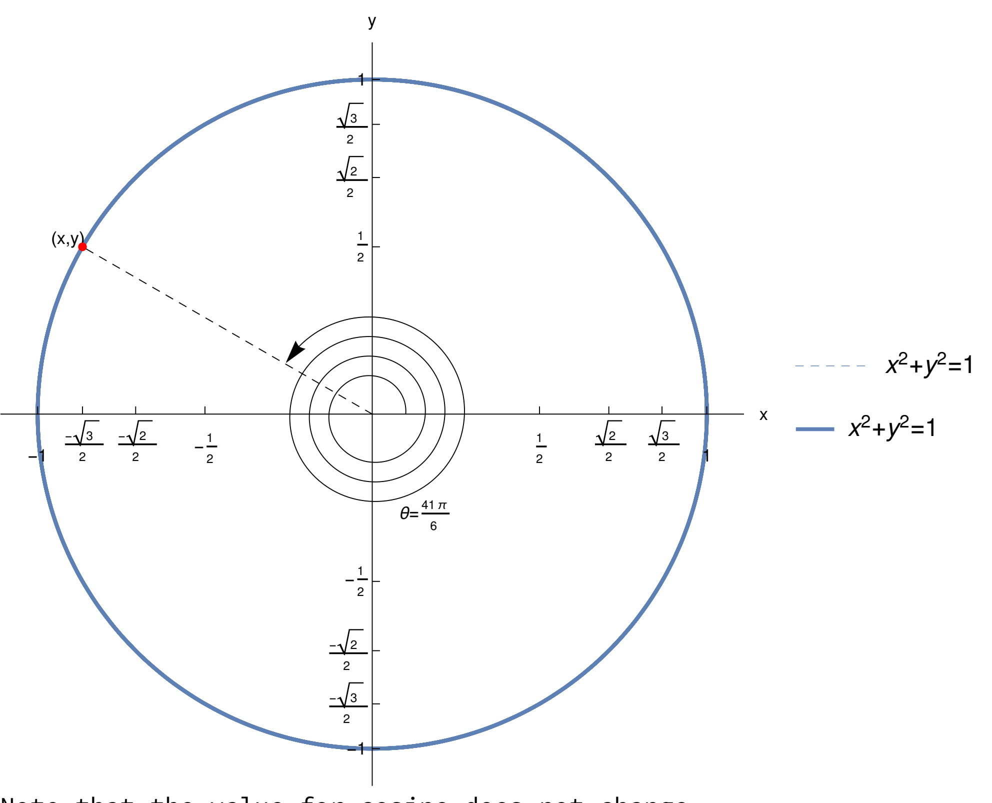
$\theta=2\pi\times1+\frac{5\pi}{6}=\frac{17\pi}{6}$



$\theta=2\pi\times2+\frac{5\pi}{6}=\frac{29\pi}{6}$

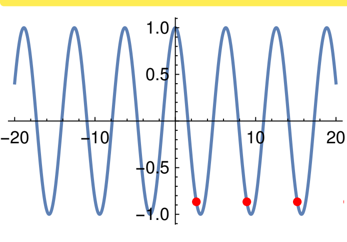


$\theta=2\pi\times3+\frac{5\pi}{6}=\frac{41\pi}{6}$



Note that the value for cosine does not change.

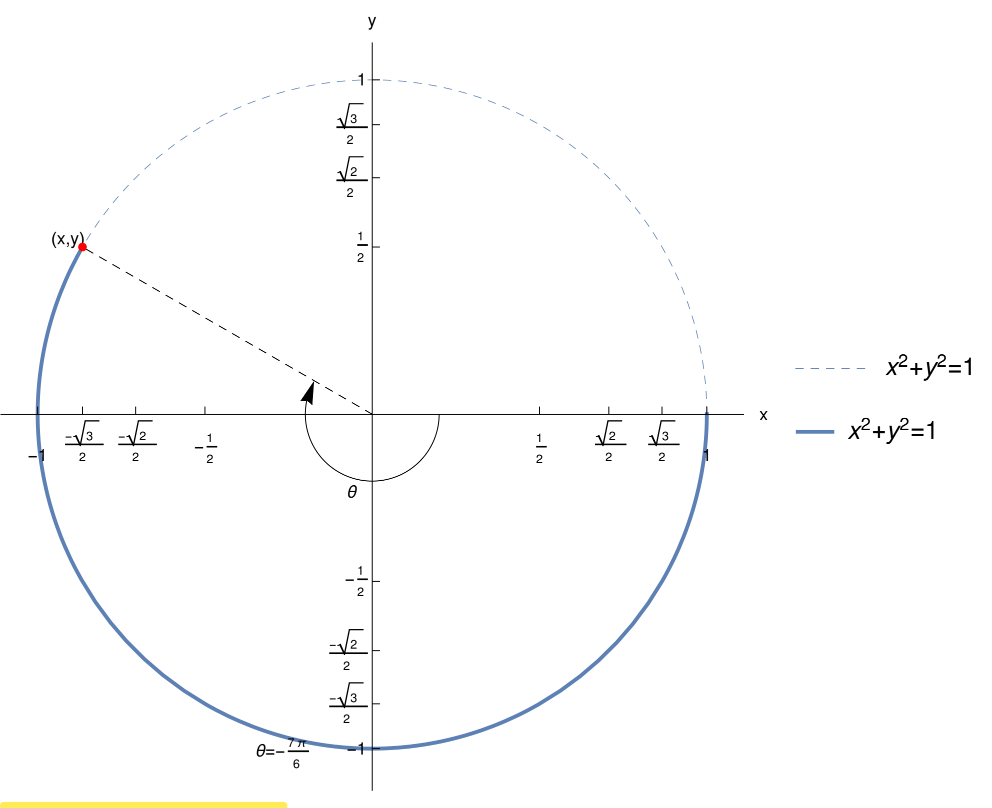
$\cos\left(\frac{5\pi}{6}+2\pi m\right)=-\frac{\sqrt{3}}{2}$,   m=0, 1, 2, ⋯ $m\in\mathbb{Z}^+$



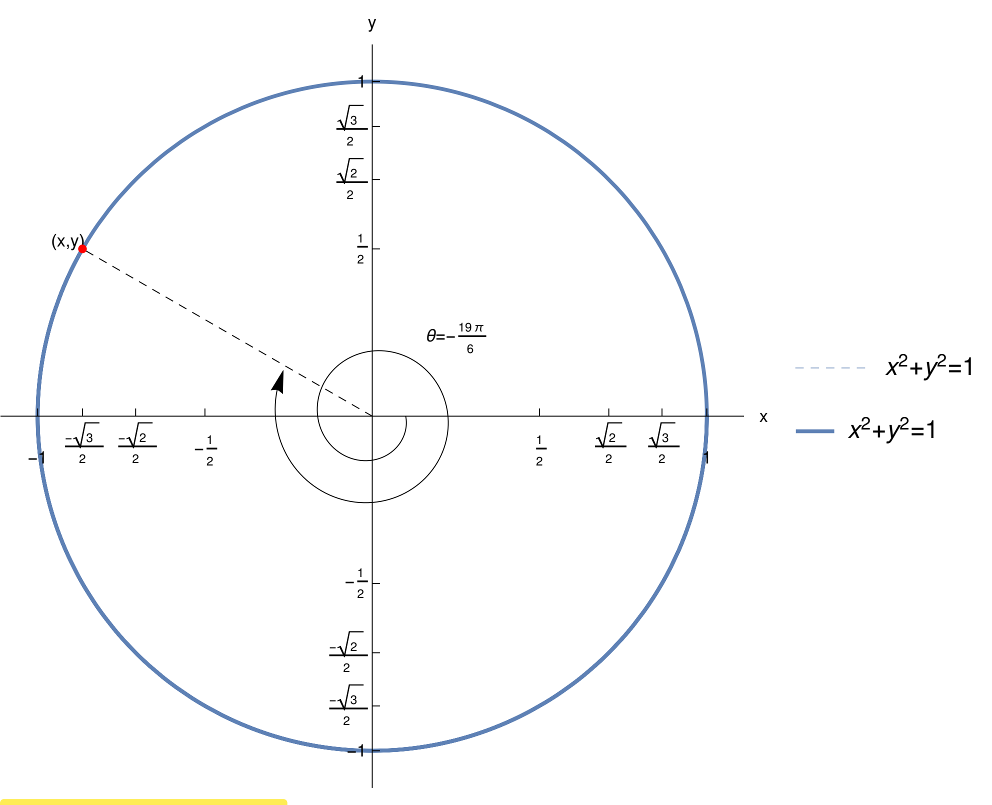
Step 3.

Add negative integer multiples of 2π

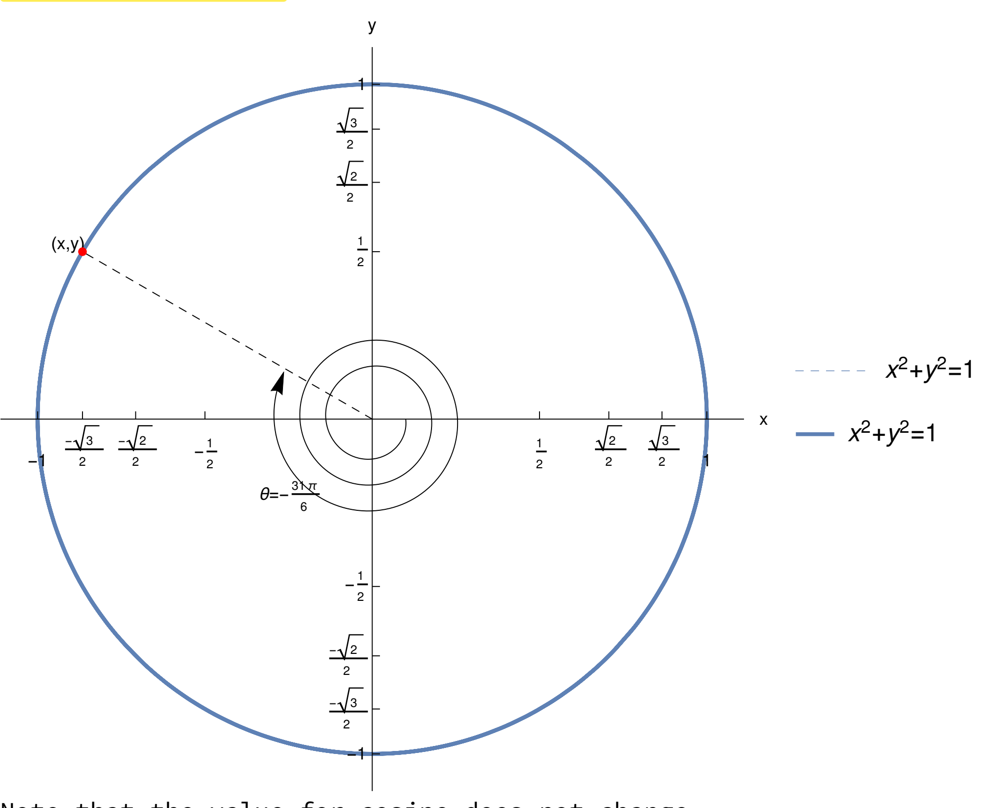
$\theta=2\pi\times(-1)+\frac{5\pi}{6}=-\frac{7\pi}{6}$



$\theta=2\pi\times(-2)+\frac{5\pi}{6}=-\frac{19\pi}{6}$



$\theta=2\pi\times(-3)+\frac{5\pi}{6}=-\frac{31\pi}{6}$

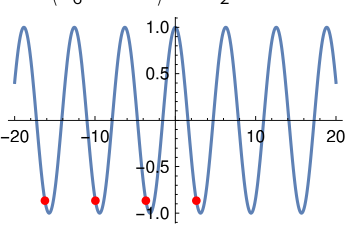


Note that the value for cosine does not change.

$\cos\left(\frac{5\pi}{6}+2\pi m\right)=-\frac{\sqrt{3}}{2}$,   m=-1, -2, ⋯ $m\in\mathbb{Z}^-$

or

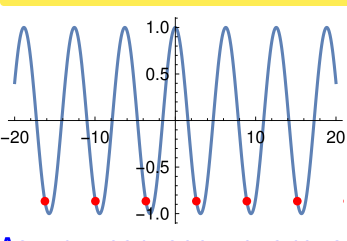
$\cos\left(\frac{5\pi}{6}-2\pi m\right)=-\frac{\sqrt{3}}{2}$,   m=1, 2, ⋯



Step 4.

Combine the two solutions

$\cos\left(\frac{5\pi}{6}+2\pi m\right)=-\frac{\sqrt{3}}{2}$,   m=⋯, -1, -2, 0, 1, 2 ⋯ $m\in\mathbb{Z}$



As you can see we are still missing half the solutions!

Step 5. Cos(-θ)=a

Repeat the above steps but multiply the starting angle by -1:

$\theta=-\left(\frac{5\pi}{6}\right)=-\frac{5\pi}{6}$

$\cos\left(-\frac{5\pi}{6}+2\pi m\right)=-\frac{\sqrt{3}}{2}$,   m=⋯, -1, -2, 0, 1, 2 ⋯ $m\in\mathbb{Z}$