Solve Sin(θ)=a for θ

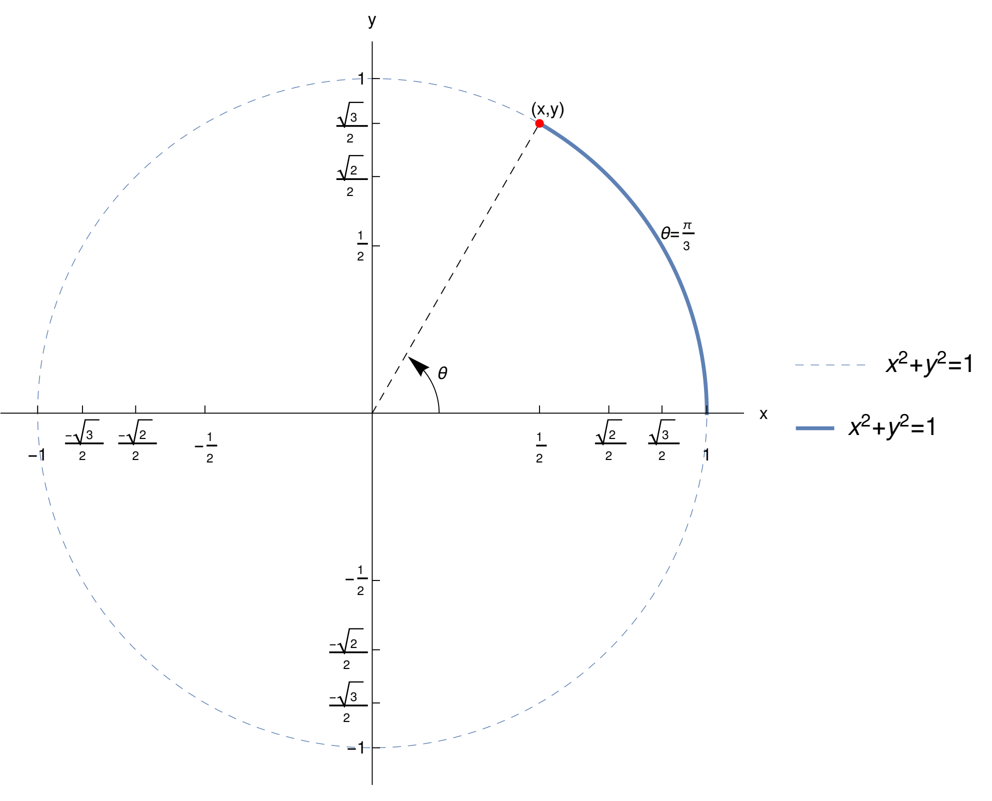
Solve trigonometric equations without using any identities.

Find all θ satisfying the equation $\sin(\theta)=\frac{\sqrt{3}}{2}$.

Step 1.

Find a known angle with the same sine value

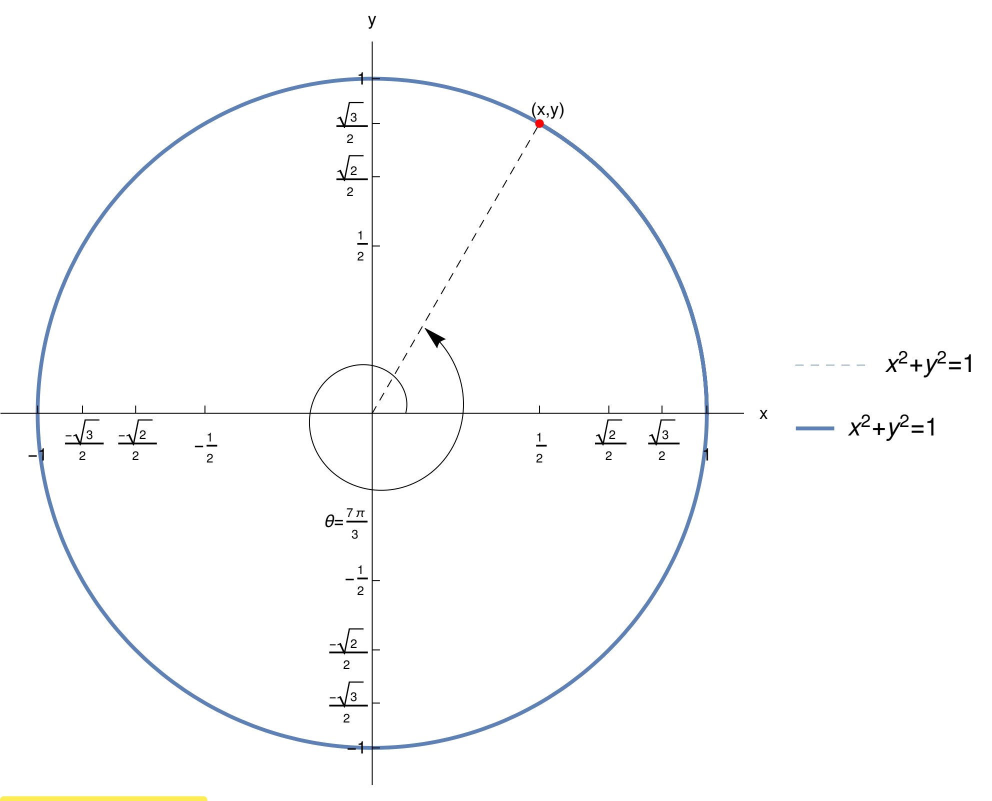
$\theta=\frac{\pi}{3}$



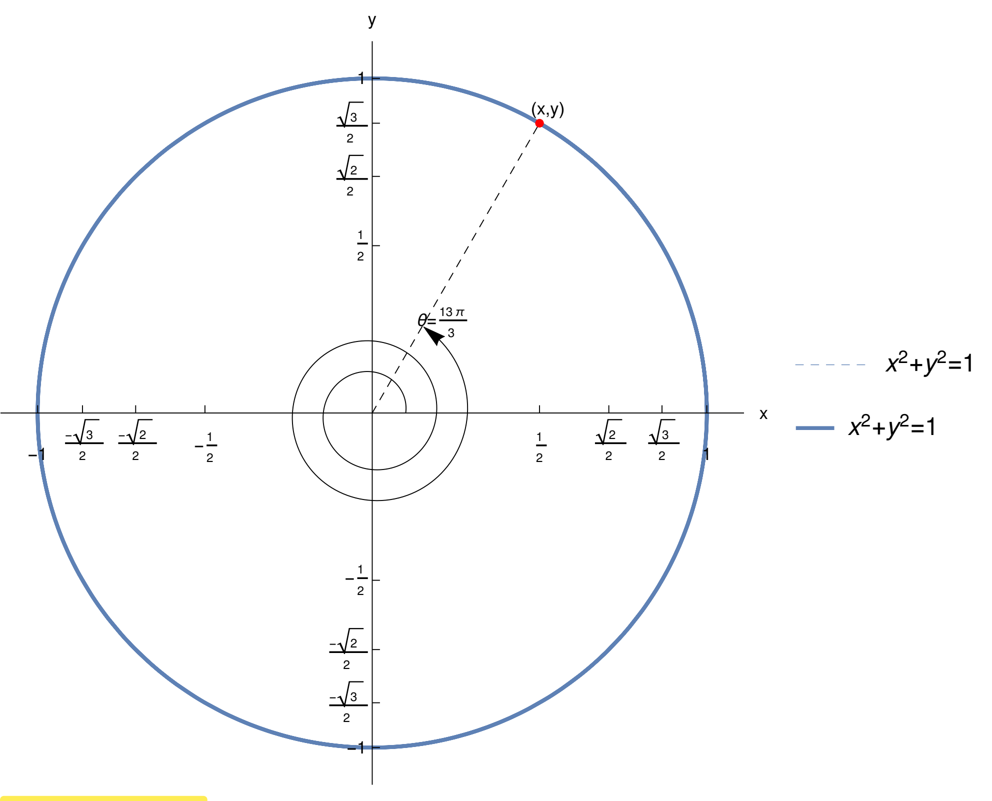
Step 2.

Add positive integer multiples of 2π

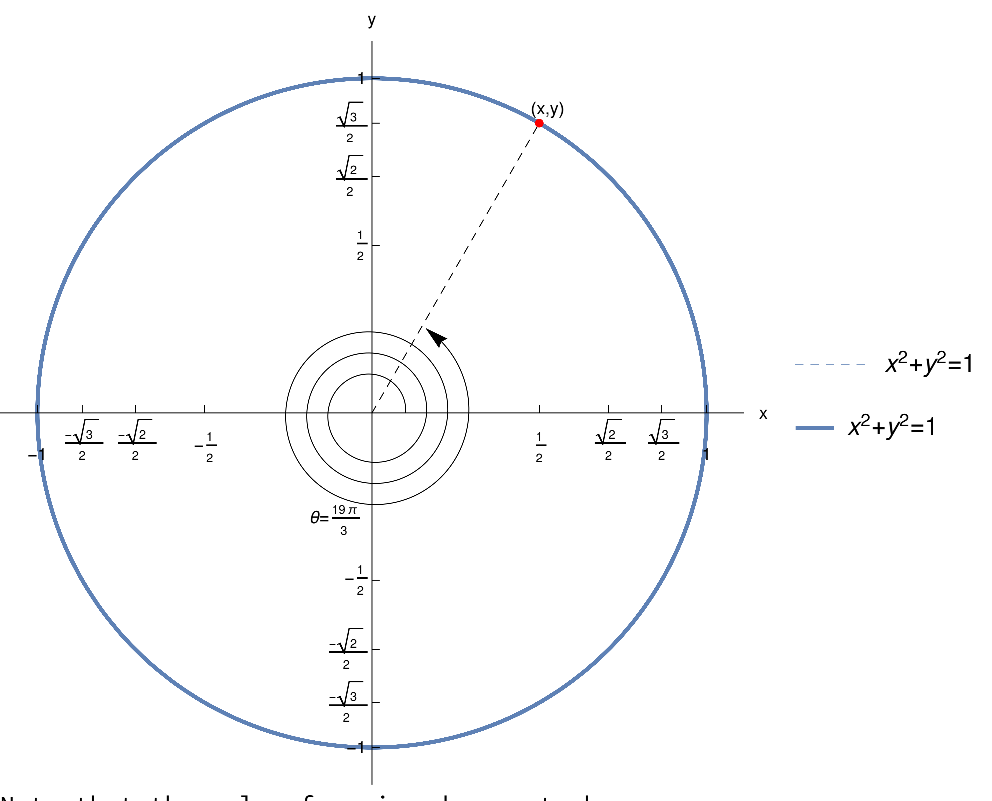
$\theta=2\pi\times1+\frac{\pi}{3}=\frac{7\pi}{3}$



$\theta=2\pi\times2+\frac{\pi}{3}=\frac{13\pi}{3}$

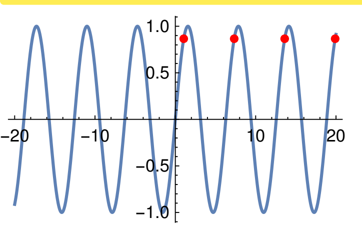


$\theta=2\pi\times3+\frac{\pi}{3}=\frac{19\pi}{3}$



Note that the value for sine does not change.

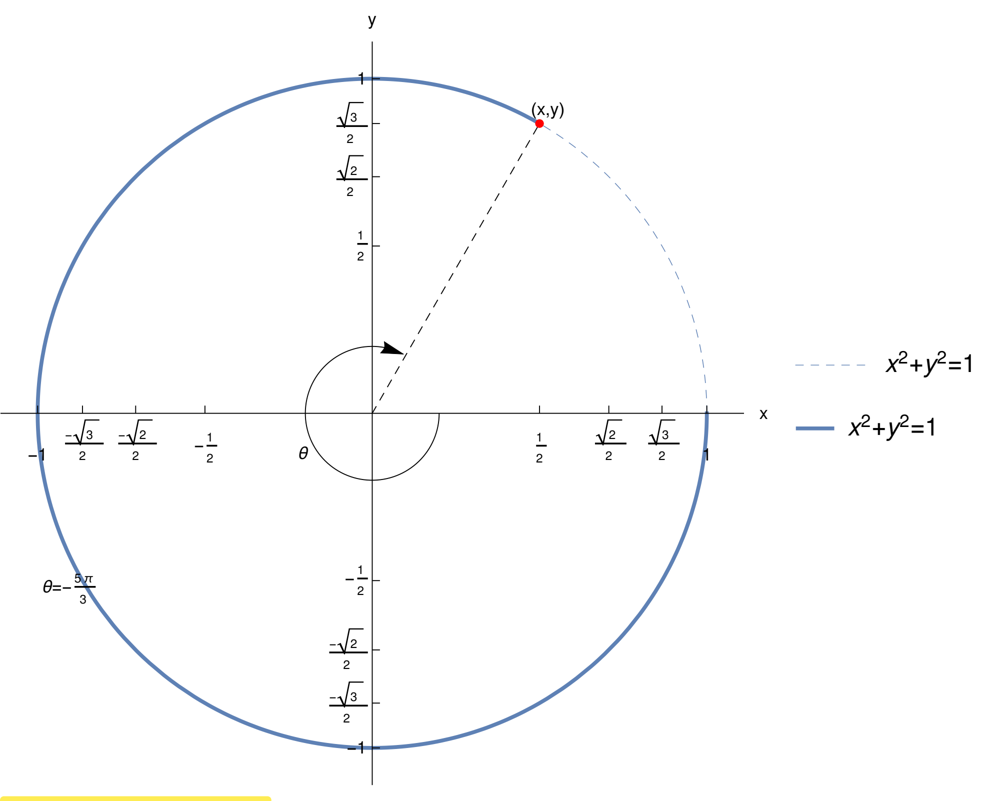
$\sin\left(\frac{\pi}{3}+2\pi n\right)=\frac{\sqrt{3}}{2}$, n=0, 1, 2, ⋯ $n\in\mathbb{Z}^+$



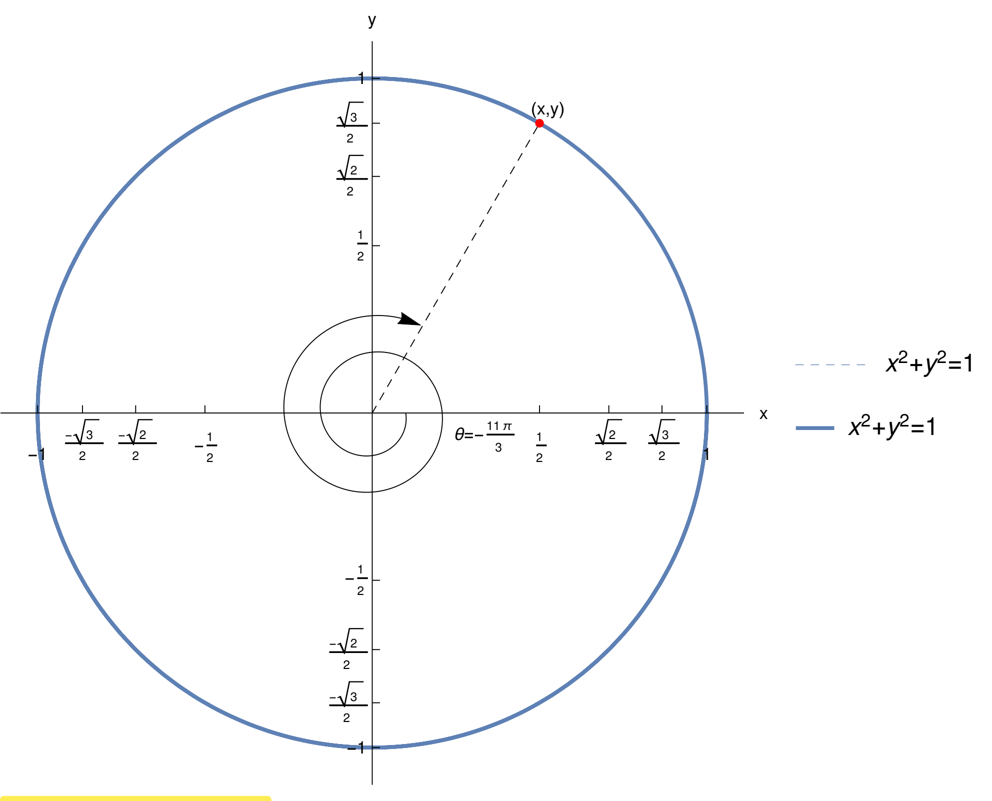
Step 3.

Add negative integer multiples of 2π

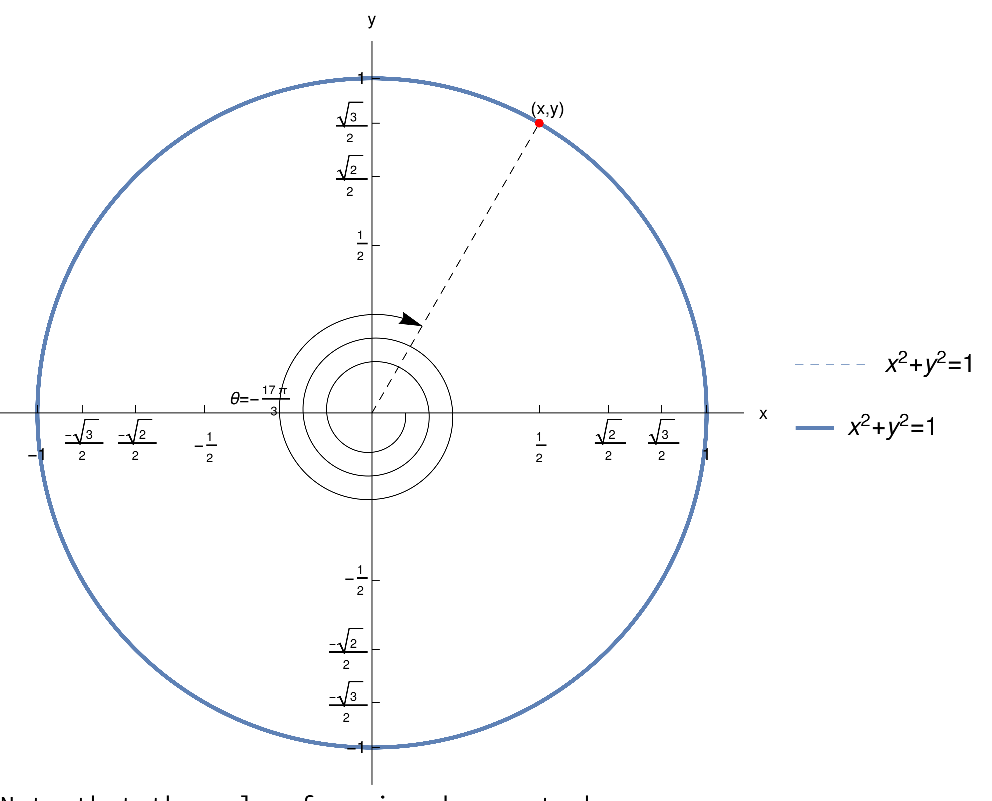
$\theta=2\pi\times(-1)+\frac{\pi}{3}=-\frac{5\pi}{3}$



$\theta=2\pi\times(-2)+\frac{\pi}{3}=-\frac{11\pi}{3}$



$\theta=2\pi\times(-3)+\frac{\pi}{3}=-\frac{17\pi}{3}$

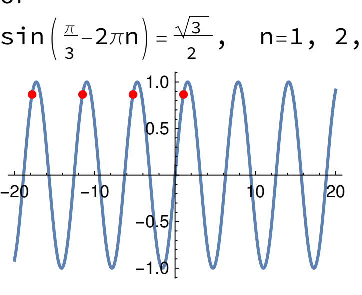


Note that the value for sine does not change.

$\sin\left(\frac{\pi}{3}+2\pi n\right)=\frac{\sqrt{3}}{2}$, n=−1, −2, ⋯ $n\in\mathbb{Z}^-$

or

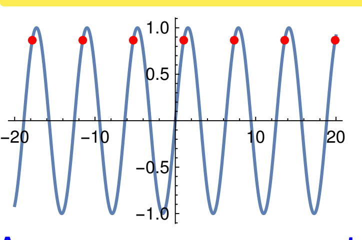
$\sin\left(\frac{\pi}{3}-2\pi n\right)=\frac{\sqrt{3}}{2}$, n=1, 2, ⋯



Step 4.

Combine the two solutions

$\sin\left(\frac{\pi}{3}+2\pi n\right)=\frac{\sqrt{3}}{2}$, n=⋯, −1, −2, 0, 1, 2 ⋯ $n\in\mathbb{Z}$



As you can see we are still missing half the solutions!

Step 5. Sin(π−θ)=a

Repeat the above steps but multiply the angle θ by −1 and add π:

$\theta=-\frac{\pi}{3}+\pi=\frac{2\pi}{3}$

Note

$\sin\left(-\frac{\pi}{3}+\pi\right)=\frac{\sqrt{3}}{2}$

$\cos\left(\frac{2\pi}{3}+2\pi(n+1)\right)=\frac{\sqrt{3}}{2}$, n=⋯, −1, −2, 0, 1, 2 ⋯ $n\in\mathbb{Z}$