Solve $\mathrm{Sin}(\theta)=a$ for θ

Solve trigonometric equations without using any identities.

Find all θ satisfying the equation $\sin(\theta)=-\frac{\sqrt{3}}{2}$.

Step 1.

Since sine value is negative, find a known first quadrant angle with sine equal to the absolute value of the given sine or $\frac{\sqrt{3}}{2}$

Step 1.a

$\theta=\frac{\pi}{3}$ where $\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

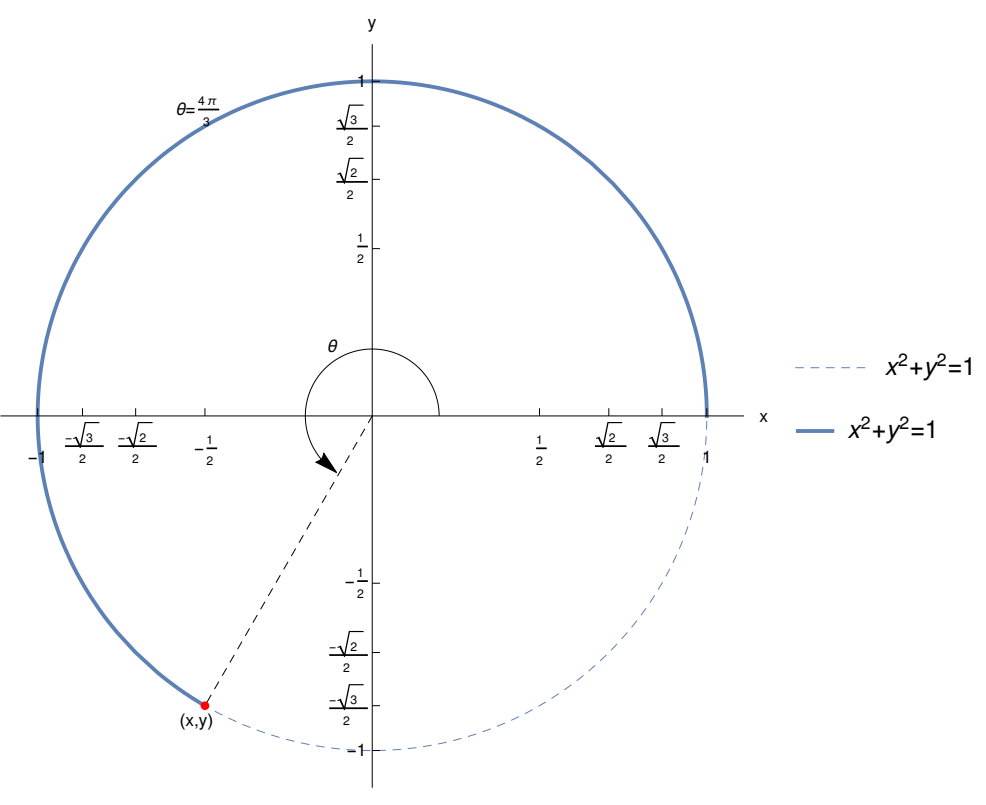
Step 1.b

Add π this angle which causes the sine to become negative

$\sin\left(\pi+\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{2}$

Switch to $\pi+\frac{\pi}{3}=\frac{4\pi}{3}$

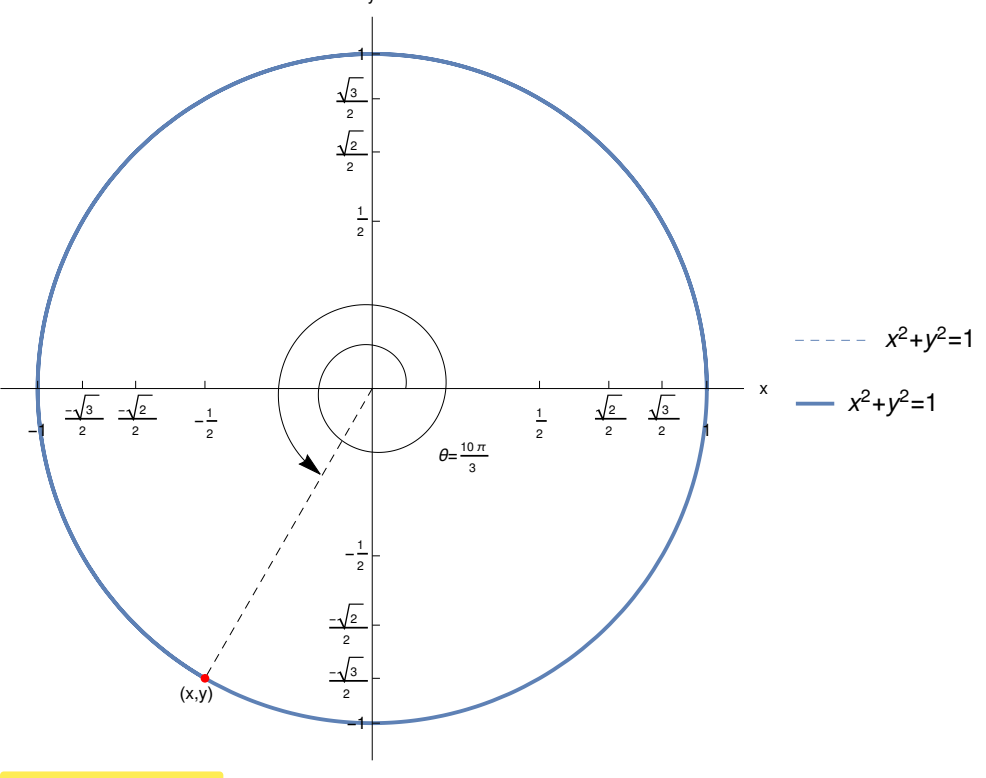
$\theta=\frac{4\pi}{3}$



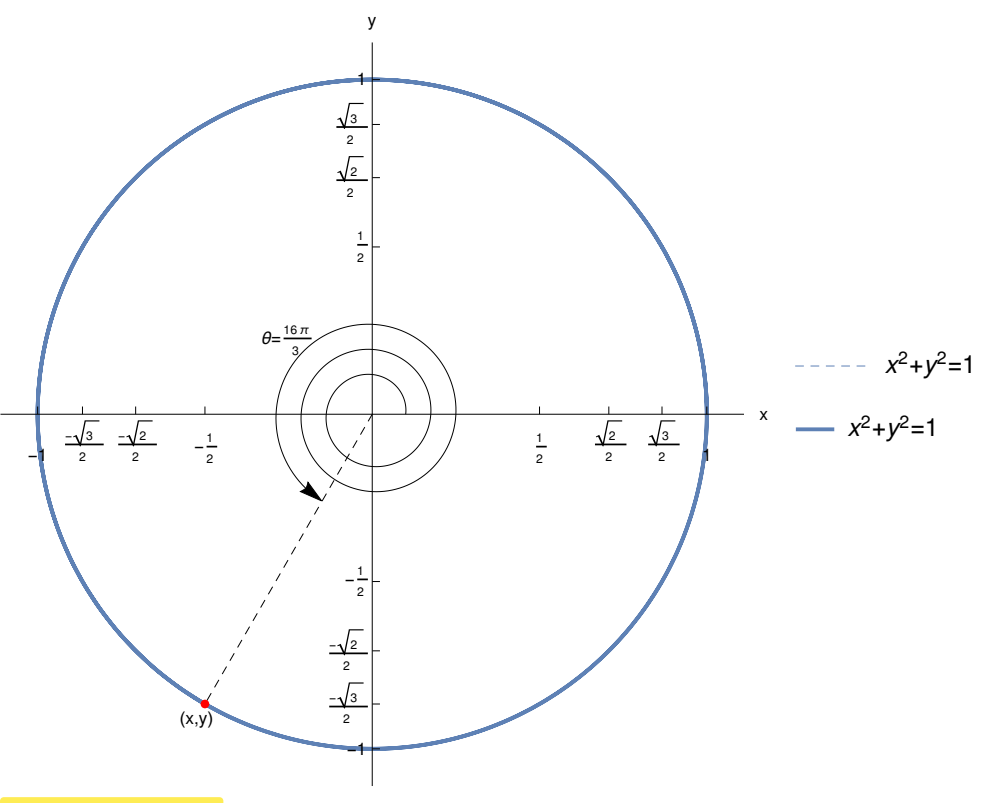
Step 2.

Add positive integer multiples of 2π

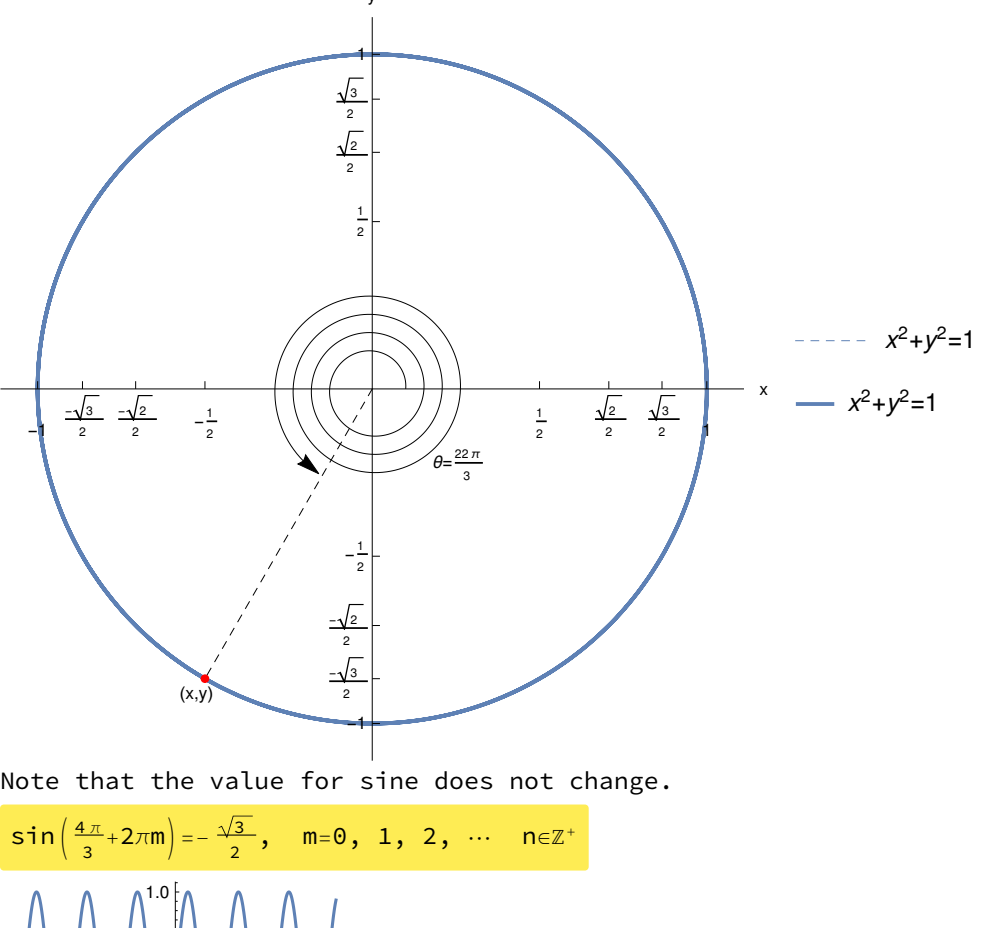
$\theta=2\pi\times1+\frac{4\pi}{3}=\frac{10\pi}{3}$



$\theta=2\pi\times2+\frac{4\pi}{3}=\frac{16\pi}{3}$

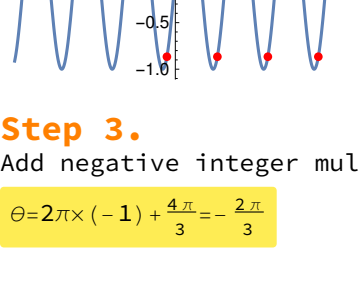


$\theta=2\pi\times3+\frac{4\pi}{3}=\frac{22\pi}{3}$



Note that the value for sine does not change.

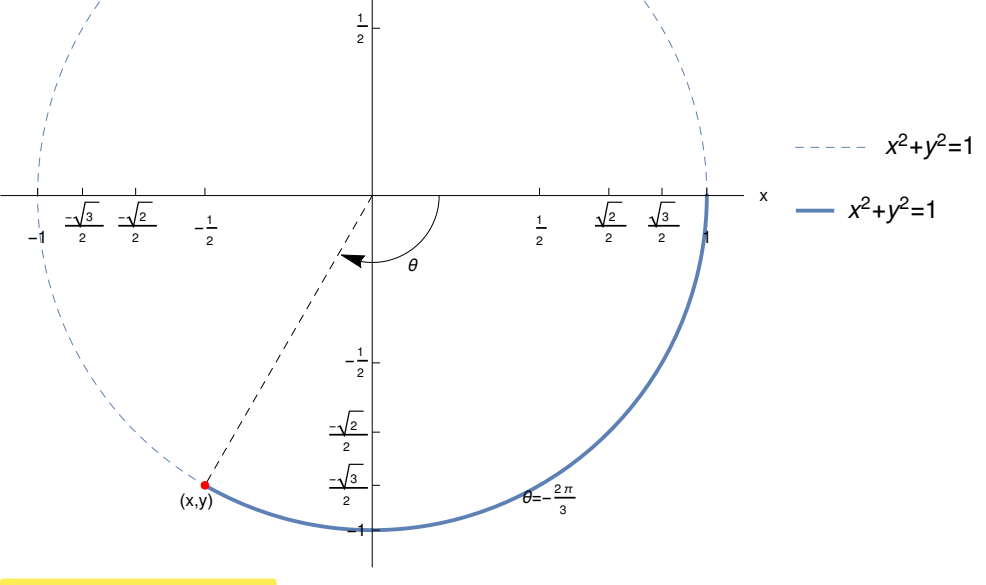
$\sin\left(\frac{4\pi}{3}+2\pi m\right)=-\frac{\sqrt{3}}{2},\quad m=0,\ 1,\ 2,\ \cdots\quad n\in\mathbb{Z}^+$



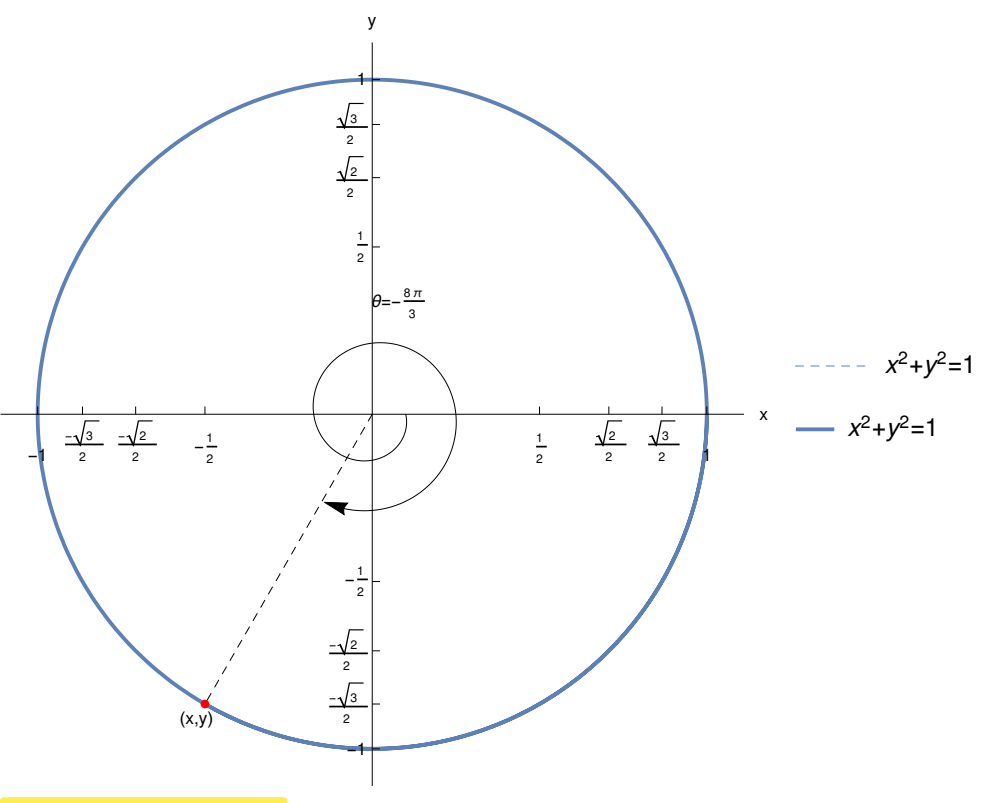
Step 3.

Add negative integer multiples of 2π

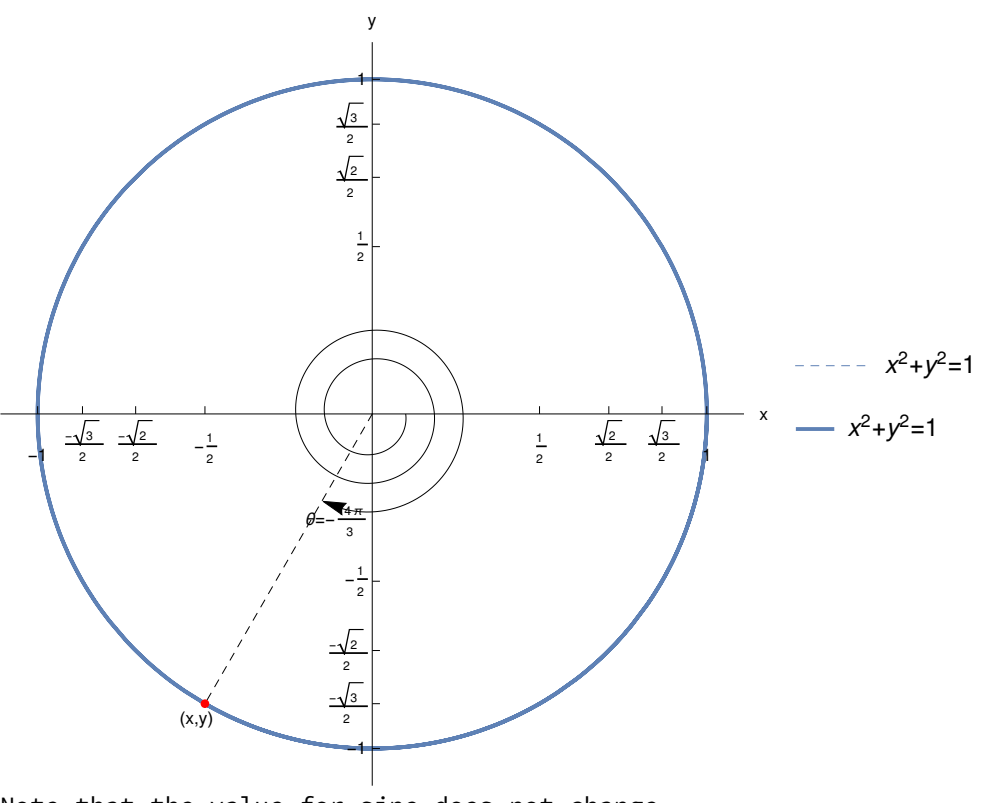
$\theta=2\pi\times(-1)+\frac{4\pi}{3}=-\frac{2\pi}{3}$



$\theta=2\pi\times(-2)+\frac{4\pi}{3}=-\frac{8\pi}{3}$



$\theta=2\pi\times(-3)+\frac{4\pi}{3}=-\frac{14\pi}{3}$

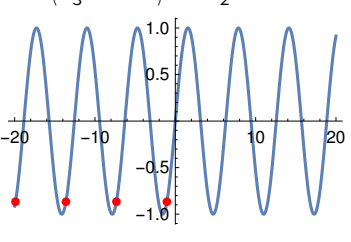


Note that the value for sine does not change.

$\sin\left(\frac{4\pi}{3}+2\pi m\right)=-\frac{\sqrt{3}}{2},\quad m=-1,\ -2,\ \cdots\quad n\in\mathbb{Z}^-$

or

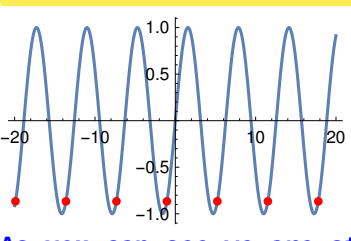
$\sin\left(\frac{4\pi}{3}-2\pi m\right)=-\frac{\sqrt{3}}{2},\quad m=1,\ 2,\ \cdots$



Step 4.

Combine the two solutions

$\sin\left(\frac{4\pi}{3}+2\pi m\right)=-\frac{\sqrt{3}}{2},\quad m=\cdots,\ -1,\ -2,\ 0,\ 1,\ 2\ \cdots\quad n\in\mathbb{Z}$



As you can see we are still missing half the solutions!

Step 5. $\mathrm{Sin}(\pi-\theta)=a$

Repeat the above steps but multiply the angle θ by -1 and add π:

$\theta=-\frac{4\pi}{3}+\pi=-\frac{\pi}{3}$

Note

$\sin\left(-\frac{4\pi}{3}+\pi\right)=-\frac{\sqrt{3}}{2}$

$\cos\left(-\frac{\pi}{3}+2\pi(m+1)\right)=-\frac{\sqrt{3}}{2},\quad m=\cdots,\ -1,\ -2,\ 0,\ 1,\ 2\ \cdots\quad m\in\mathbb{Z}$