Solve Sin(θ)=a for θ

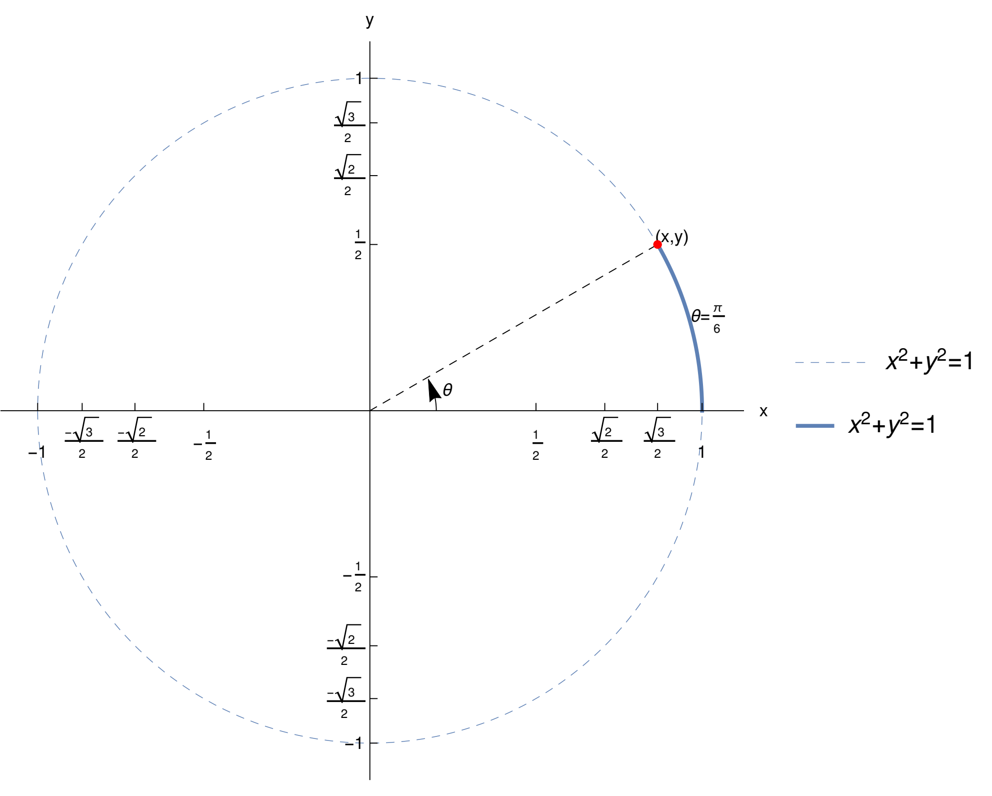
Solve trigonometric equations without using any identities.

Find all θ satisfying the equation $\sin(\theta)=\frac{1}{2}$.

Step 1.

Find a known angle with the same sine value

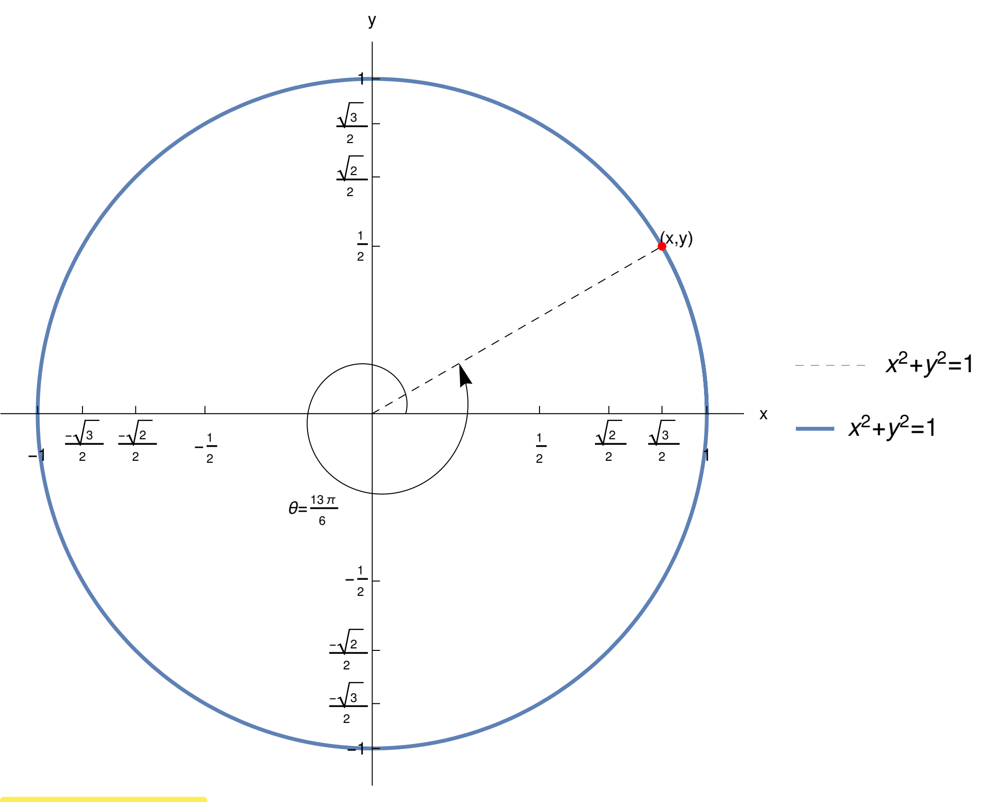
$\theta=\frac{\pi}{6}$



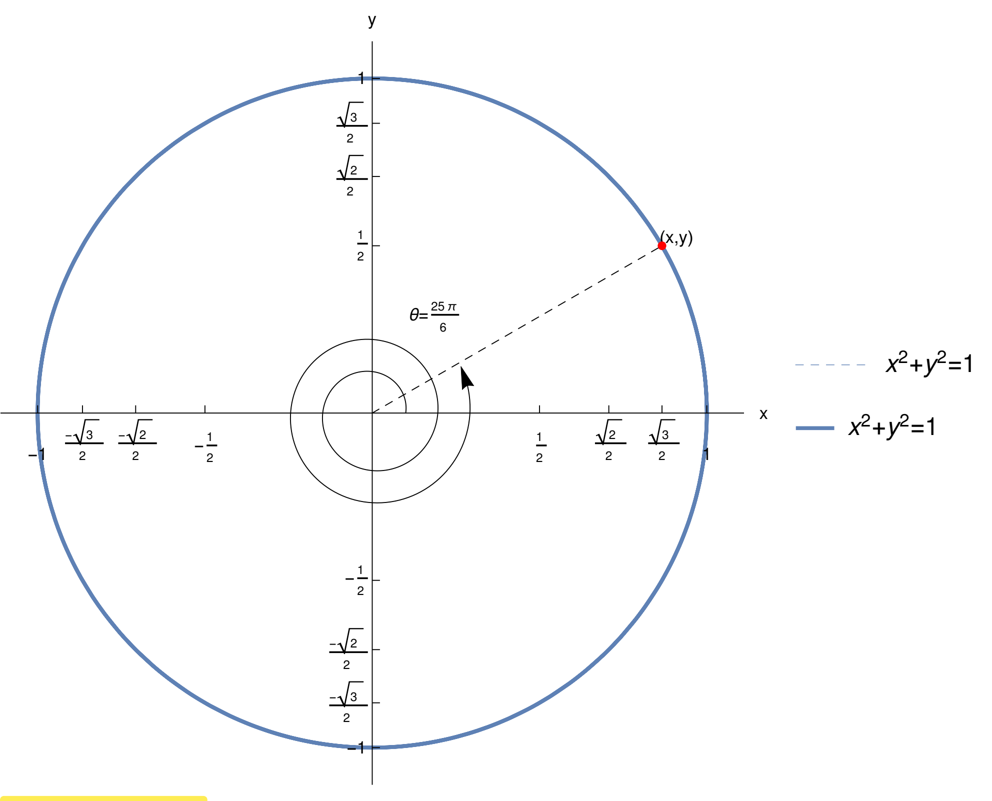
Step 2.

Add positive integer multiples of 2π

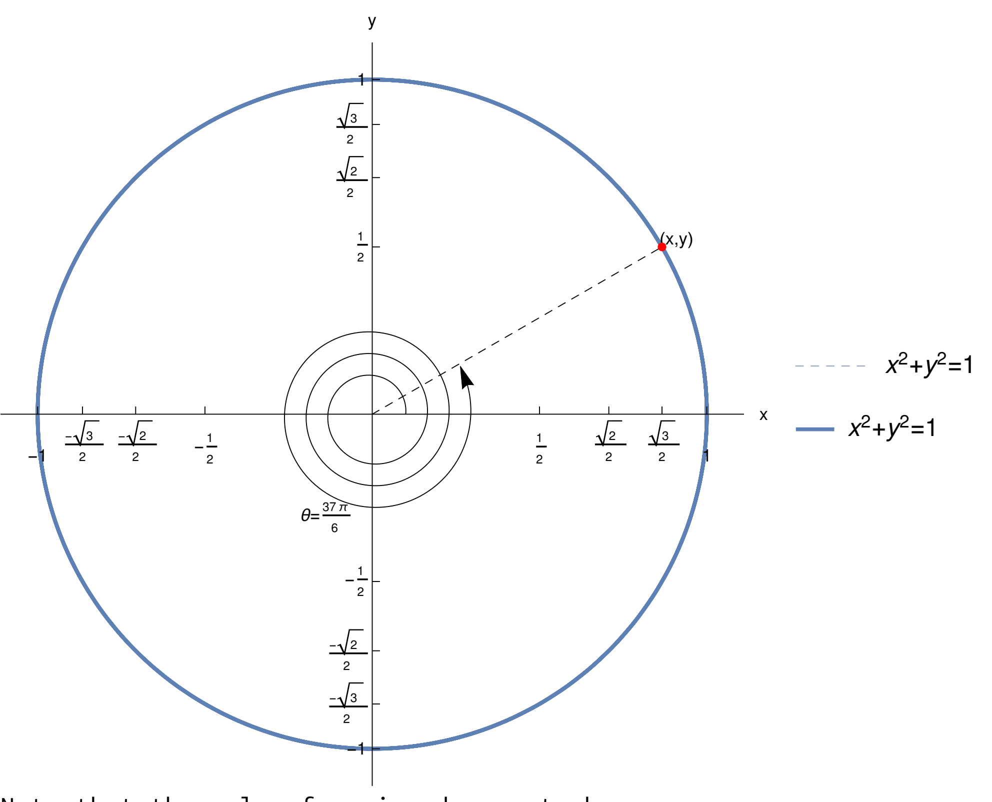
$\theta=2\pi\times 1+\frac{\pi}{6}=\frac{13\pi}{6}$



$\theta=2\pi\times 2+\frac{\pi}{6}=\frac{25\pi}{6}$

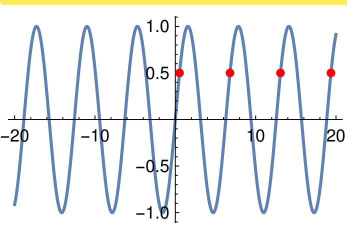


$\theta=2\pi\times 3+\frac{\pi}{6}=\frac{37\pi}{6}$



Note that the value for sine does not change.

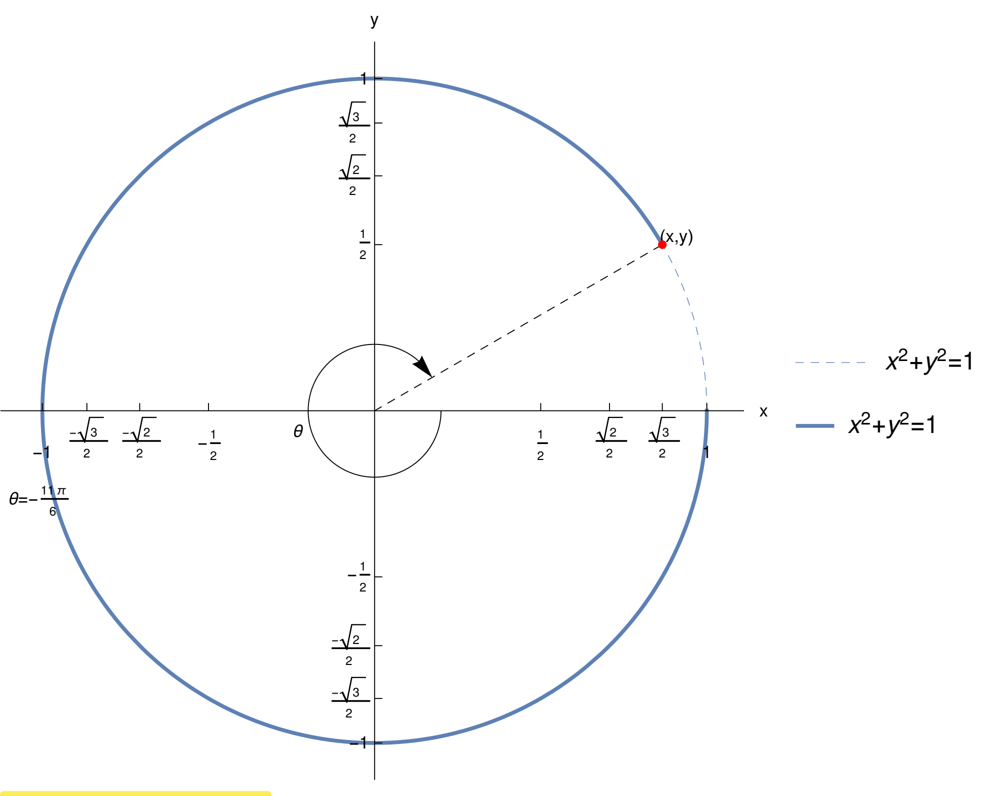
$\sin\left(\frac{\pi}{6}+2\pi n\right)=\frac{1}{2}$,   n=0, 1, 2, ⋯ $n\in\mathbb{Z}^+$



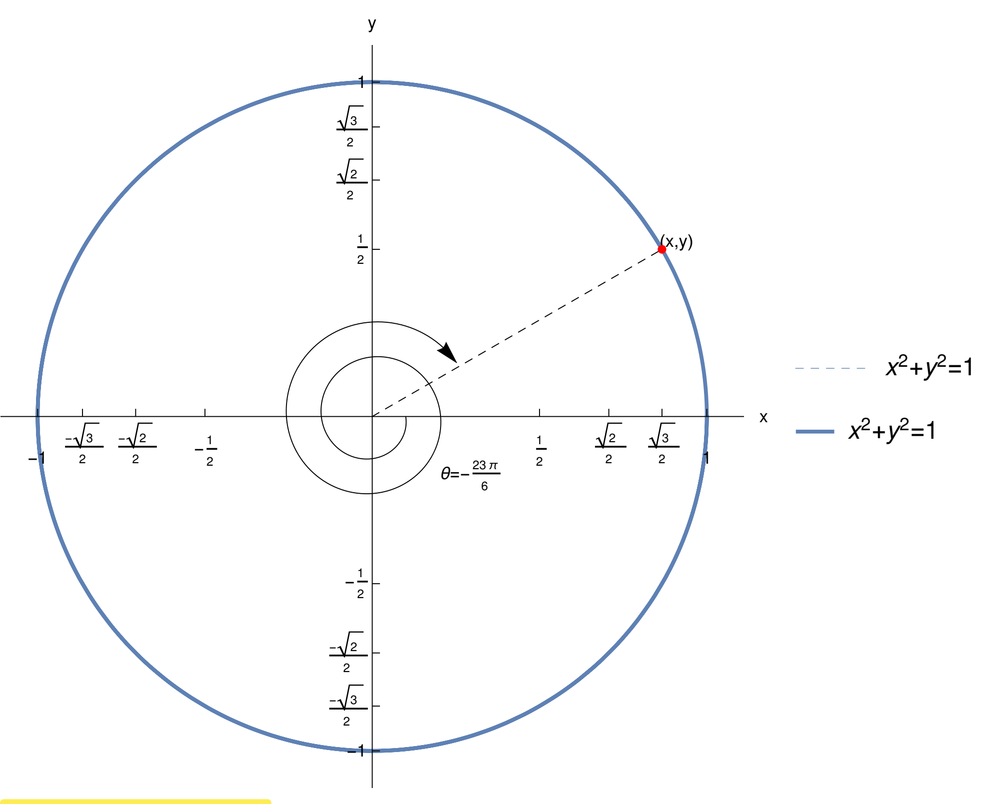
Step 3.

Add negative integer multiples of 2π

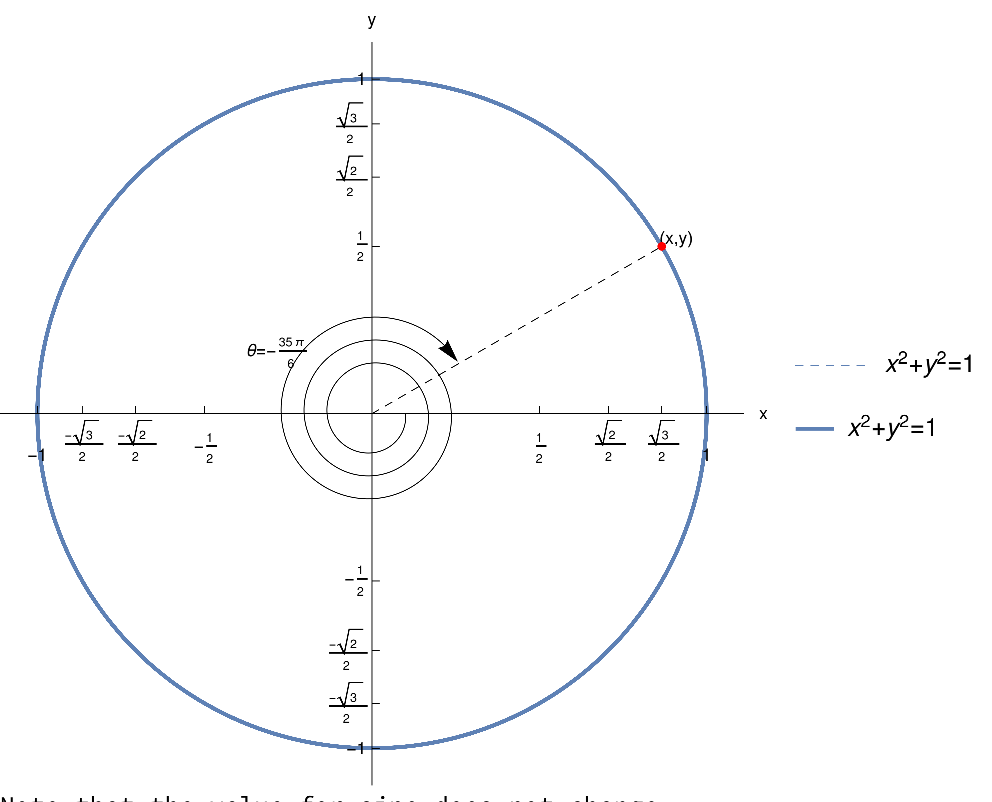
$\theta=2\pi\times(-1)+\frac{\pi}{6}=-\frac{11\pi}{6}$



$\theta=2\pi\times(-2)+\frac{\pi}{6}=-\frac{23\pi}{6}$



$\theta=2\pi\times(-3)+\frac{\pi}{6}=-\frac{35\pi}{6}$

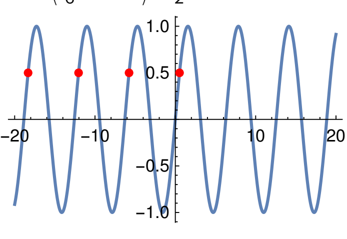


Note that the value for sine does not change.

$\sin\left(\frac{\pi}{6}+2\pi n\right)=\frac{1}{2}$,   n=-1, -2, ⋯ $n\in\mathbb{Z}^-$

or

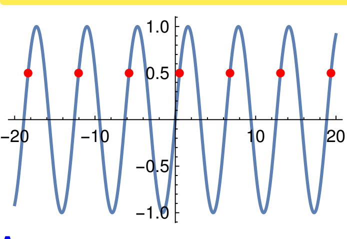
$\sin\left(\frac{\pi}{6}-2\pi n\right)=\frac{1}{2}$,   n=1, 2, ⋯



Step 4.

Combine the two solutions

$\sin\left(\frac{\pi}{6}+2\pi n\right)=\frac{1}{2}$,   n=⋯, -1, -2, 0, 1, 2 ⋯ $n\in\mathbb{Z}$



As you can see we are still missing half the solutions!

Step 5. Sin($\pi-\theta$)=a

Repeat the above steps but multiply the angle θ by -1 and add π:

$\theta=-\frac{\pi}{6}+\pi=\frac{5\pi}{6}$

Note

$\sin\left(-\frac{\pi}{6}+\pi\right)=\frac{1}{2}$

$\cos\left(\frac{5\pi}{6}+2\pi(n+1)\right)=\frac{1}{2}$,   n=⋯, -1, -2, 0, 1, 2 ⋯ $n\in\mathbb{Z}$